UNSTABLE GAS FLOW MODES IN A RANQUE VORTEX TUBE

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UDC 532.527.004.14

The phenomenon of excitation of developed low- and high-frequency regular pressure fluctuations in the vortex tube of energy separation is experimentally established.

The vortex effect of energy separation of a gas stream is extensively used in different branches of engineering [1]. A large quantity of papers [2] are devoted to its investigation, where different aspects of the flow of a helical gas stream and the heat- and mass-transfer processes therein are examined in detail in a static formulation. However, the physical nature of the vortex effect has still not received exhaustive explanation. This is related to the exceptional complexity of the energy of mass transfer processes occurring in a three-dimensional helical stream with large radial velocity pressure gradients. In practice, nothing is known about the dynamical properties of the flow in a vortex tube, meanwhile rotating streams tend to a different kind of flow instability [3, 4], and the regular pressure fluctuations being generated here cannot exert influence on the transfer processes, and therefore, on the efficiency of energy separation of the gas.

An attempt is made in this paper to investigate the condition for the origination of unstable flow modes in vortex tubes, and to store and analyze the experimental material needed to clarify, later, the role of the dynamic flow properties in realizing the vortex effect.

The investigation was performed on adiabatic vortex tubes (smooth and with a rectifying spider in the area of the choke) with the geometric dimensions: $d = 32$ mm, $\bar{d}_d = d_d/d = 0.45$, $\bar{F}_{in} = 4F_{in}/\pi d^2 = 0.1$; $\bar{L} = L/d = 4$–40. The tube length was measured by a successive set of sections of seamless brass tubes of different lengths. A three-nozzle vortex generator with rectangular tangential channels (9×3 mm) was used for the air supply. The condition for adiabaticity of the process was achieved by isolating the tube sections and the cold air plenum by using asbestos and glass fabric.

The pressure fluctuations in the stream were measured by using a piezotransducer whose sensor (a plate of lead zirconate of $d = 10$ mm and $b = 1$ mm) was fastened flush with the wall of the measured section of the tube. Calibration of the transducer in a powerful sound field showed that its amplitude–frequency characteristic is linear, and the calibration factor is 0.0405 mV/Pa. An array of secondary apparatus, consisting of a two-channel oscilloscope C1-18, a millivoltmeter V3-28A, a harmonic analyzer C5-3, a PSG-101 recorder from the firm RFT (GDR), permitted an amplitude–frequency analysis of the signal from the pressure transducer in the 50–20,000-Hz band.

Simultaneous measurement of the hydrodynamic and thermodynamic characteristics of the vortex tubes with the rectifying spider showed that the flow in the tube is unstable in the whole range of operating modes (Fig. 1). Analysis of the pressure fluctuations spectra permits the assertion that the presence of three kinds of fluctuations is characteristic for vortex tubes: white (wideband) background noise of turbulent origin, low-frequency (l.f.) periodic pressure fluctuations at the frequency $f_{l.f.} = 1000$–2000 Hz, and high-frequency (h.f.) periodic pressure fluctuations with $f_{h.f.} = 12{,}000$–$18{,}000$ Hz. The low-frequency fluctuations yield the main contribution to the energy spectrum of the pressure pulsations. Their amplitude ($\Delta P'_{l.f.}$) grows monotonically as μ increases to $\mu = 0.9$–0.95 and then jumps an order of magnitude and reaches the maximum value at $\mu = 1$ (Fig. 1). The jump increase in $\Delta P'_{l.f.}$ is accompanied by a substantial rise (by 8–10%) in the hydraulic drag of the vortex tube. A large quantity of l.f. fluctuation subharmonics hence appears in the pressure fluctuation spectrum, which also explains the significant difference between the quantities $\Delta P'_{l.f.}$ and $\Delta P'_{\Sigma}$ at $\mu > 0.9$–0.95.

High-frequency pressure fluctuations with large amplitude are observed only after the jump in hydraulic drag, the h.f. fluctuations are unstable in the range of values $\mu = 0.6$–0.9 (the h.f. signal generally drops out from time to time) and their amplitude is small.

Regular low- and high-frequency pressure fluctuations appear in the vortex tube for $\Delta P \geq 40$ kPa. As the pressure drops grow, the fluctuation frequency rises continuously: an increase in ΔP of 40–400 kPa corresponds to a rise in $f_{l.f.}$ of 1100–1850 Hz and in $f_{h.f.}$ of 11,750–13,260 Hz ($\bar{L} = 9.5$ and $\mu = 1$).

Kuibyshev Aviation Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 41, No. 5, pp. 784–790, November, 1981. Original article submitted April 17, 1980.

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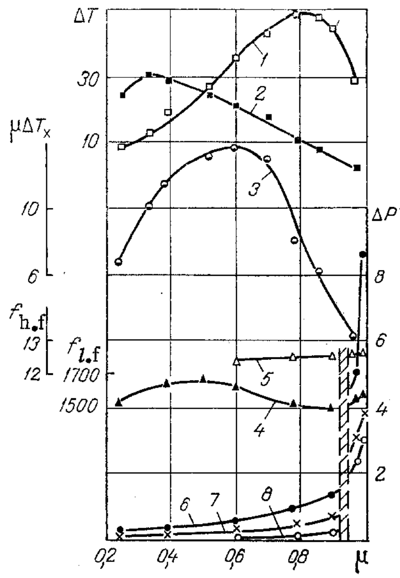

Fig. 1. Dependence on the relative mass flow rate of the cold air: 1) ΔT_g, °K; 2) ΔT_x, °K; 3) $\mu\Delta T_x$, °K; 4) $f_{l.f.}$, Hz; 5) $f_{h.f.}$, kHz; 6) $\Delta P'_{\Sigma}$, kPa; 7) $\Delta P'_{l.f.}$, kPa; 8) $\Delta P'_{h.f.}$, kPa; $\bar{L} = 9.5$; $\bar{L}_d = 7.5$; $\Delta P = 200$ kPa.

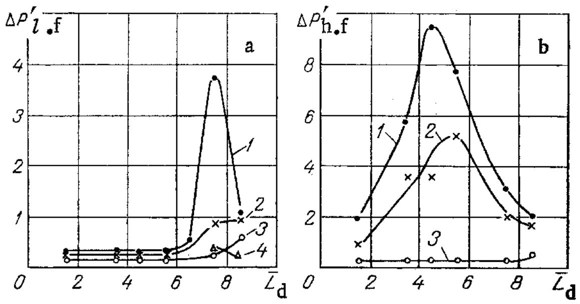

Fig. 2 Change in amplitude of the low-frequency (a) and high-frequency pressure fluctuations (kPa) along the vortex tube length with $\bar{L} = 9.5$ for $\Delta P = 200$ KPa: 1) $\mu = 1$; 2) 0.97; 3) 0.83; 4) 0.28.

The quantity $\Delta P'_{l.f.}$ is practically unchanged in the initial section of the tube and starts to rise intensively in the choke direction only in the last 3-4 calibers (Fig. 2a). In the range $\mu = 0.95$-1 (i.e., after the jump in the hydraulic drag), $\Delta P'_{l.f.}$ and $\Delta P'_{h.f.}$ have quite definite maximums shifted 2 and 5 calibers deeply into the tube, respectively (Fig. 2a and b). The frequencies of both signals do not vary along the tube length.

A change in the vortex tube length in the range $\bar{L} = 4.5$-18.5 exerts no influence on the qualitative nature of the dependences $\Delta P'_{l.f.}$, $\Delta P'_{h.f.}$, $f_{l.f.}$, $f_{h.f.} = f(\mu)$. The maximum *l.f.* and h.f. fluctuation intensity occurs at $\bar{L} = 9.5$. Let us note that the greatest cold productivity in a vortex tube of the type under consideration is observed at this value of $\bar{L}$ [1]. An increase in the vortex tube length has practically no influence on the quantity $f_{h.f.}$ but results in a substantial drop in $f_{l.f.}$. Thus, for $\mu = 1$ and $\Delta P = 200$ kPa, an increase in $\bar{L}$ from 4.5 to 18.5 calibers will reduce $f_{l.f.}$ from 2000 to 1200 Hz.

Most characteristic for smooth vortex tubes (without rectifying spiders) are the h.f. pressure fluctuations. The appearance of intensive h.f. fluctuations occurs in a jump and depends on $\bar{L}$, μ and ΔP; the smaller the values of $\bar{L}$ and μ, the greater magnitudes of ΔP needed to develop a powerful fluctuation process. In our experiments ($\Delta P = 100$-300 kPa, $\bar{L} = 4$-40), the time of the appearance of intensive fluctuations corresponded to the values $\mu = 1.0$-0.7.

Analysis of the development process of h.f. instability in the example of a tube with $\bar{L} = 6.5$ showed that for $\Delta P < 110$ kPa, $\Delta P'_{h.f.}$ is at the turbulent noise level ($\Delta P'_{\Sigma} = 338$ Pa, $\Delta P'_{h.f.} = 50$ Pa). For $\Delta P = 110$ kPa a jump (40-fold) increase in the h.f. signal amplitude occurs. The main fraction of the pressure fluctuation intensity is here in the discrete component with $f_{h.f.} = 17{,}450$ Hz ($\Delta P'_{\Sigma} = 2500$ Pa, $\Delta P'_{17{,}450} = 2000$ Pa) and a small fraction is in the discrete component with $f = 12{,}350$ Hz ($\Delta P'_{12{,}350} = 150$ Pa). An increase in ΔP to 140 kPa results in a second jump (fourfold) in the rise of the fluctuation amplitude. Only one discrete component with $f = 12{,}650$ Hz ($\Delta P'_{\Sigma} = 10.75$ kPa, $\Delta P'_{12{,}650} = 8.75$ kPa).hence occurs in the spectrum. As the pressure drop increases further to 400 kPa, the quantities $\Delta P'_{h.f.}$and $f_{h.f.}$ grow monotonically. Therefore, an insignificant increase in ΔP (100-140 kPa) results in a two order-of-magnitude rise in the h.f. fluctuation amplitude. Growth of $\Delta P'_{h.f.}$ occurs in two jumps with the presence of an intermediate fluctuation frequency (17,450 Hz) and is not accompanied by a change in the hydraulic drag of the tube.

A jump change in ΔP^{Σ} is shown in Fig. 3 for short tubes of different length. In long tubes ($\bar{L} = 20$-40), the pressure fluctuation intensity is an order of magnitude lower than in short tubes. Thus, for $\bar{L} = 32$ we have $\Delta P'_{\Sigma} = 1600$ Pa, while $\Delta P'_{13{,}200} = 450$ Pa. The pressure fluctuation spectrum becomes complex here and contains a large number of discrete components, including low-frequency components.

Measurements of the pressure fluctuations at different sections of the vortex tubes showed that the frequency fluctuation is constant along the tube length, while the amplitude grows monotonically toward the choke, where the most intensive growth of $\Delta P'_{h.f.}$ for any values of $\bar{L}$ and μ is observed in the last 4-5 calibers.

Analysis of the frequency and amplitude dependences of the low-frequency fluctuations on ΔP, $\bar{L}$ and $\bar{L}_d$ exhibits their total qualitative correspondence with analogous dependences for the vortex sound generator (VSG) [4, 5], which is a short vortex tube ($\bar{L} = 2$-4) without a choke and with an endface wall instead of a diaphragm. This affords a foundation for assuming that the h.f. pressure fluctuations in a vortex tube with rectifying spider are a result of a precessional hydrodynamic instability of the three-dimensional helical gas stream whose nature was established in investigating the VSG [5]. The crux of the mechanism of such a hydrodynamic flow instability is that during transfer of the moment of momentum from a peripheral free vortex to a near-axial countercurrent, the latter deviates from the tube axis and starts to precess around it, periodically deforming the boundary of the peripheral vortex and thereby causing regular velocity and pressure fluctuations therein. The frequency of these pulsations equals the precession frequency of the stimulated vortex and is determined by the volume mass flow rate of the working body through the tube, the degree of twist of the stream (the ratio of the moment of momentum to the momentum), and the tube diameter. As the tube length increases, the fluctuation frequency drops because of the rise in the loss of the moment of momentum due to friction on the interfacial surface of the vortices. The amplitude of the pressure fluctuations in the tube is determined by the radius of precession (the magnitude of the shift) of the stimulated vortex and the level of the absolute velocities in the free vortex.

The distinctive feature of the precessional instability is its nonsymmetric (tangential) fluctuation mode; therefore, if two pressure transducers are set at diametrically opposite points of the tube, then their signals should be out of phase. Confirmation showed that the *l*.f. instability in the vortex tube has precisely such a fluctuation mode. Computed values of $f_{l.f.}$, obtained by using formulas to determine the audio frequency emitted by the VSG [4], differ by 10-30% from the experimental results. This is evidently related to the structural differences between the vortex tube and the VSG.

A sharp increase in the amplitude of the *l*.f. fluctuations for high values of μ is apparently explained by reaching the optimal relationship between the free and stimulated vortex masses for development of the fluctuation process. This relationship is achieved in the VSG for perfectly definite values of $\bar{L}$, to which the maximum rarefaction of the tube axis, and therefore, the most intensive countercurrent, will correspond [5].

A jump increase in the hydraulic drag during excitation of the developed low-frequency fluctuations also indicates the precessional nature of the *l*.f. instability. It is shown in [6] that sudden excitation of precessional fluctuations in a vortex tube (VSG) results in a 10-20% upward jump in the tube hydraulic drag. Such an increase is most often associated with elevated losses in the moment of momentum of the peripheral stream, whose kinetic energy is a source of the energy sustaining the precessional autooscillations.

The nature of the flow h.f. instability is less clear. It is not a result of acoustic resonance since $f_{h.f.}$ depends quite weakly on the tube length (as $\bar{L}$ increases from 6.5 to 18.5 calibers, $f_{h.f.}$ drops by 3.5% from 13,200 to 12,780 Hz for the case $\mu = 1$ and $\Delta P = 200$ kPa), and grows monotonically as ΔP increases. The initial perturbations introduced into the flow because of the air supply to the tube through the discrete nozzles also do not affect the excitation of the h.f. instability: the quantities $\Delta P'_{h.f.}$ and $f_{h.f.}$ do not change when the three-nozzle vortex generator is replaced by a four-nozzle generator with the same value of F_{in}.

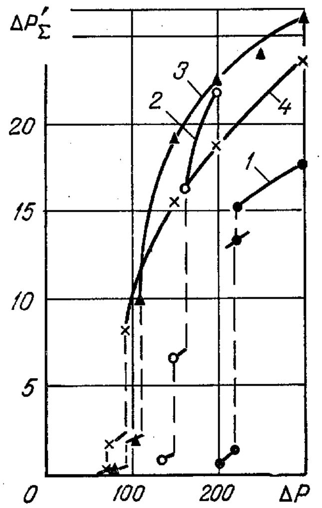

Fig. 3. Dependence of the total pressure fluctuation level $\Delta P'_{\Sigma}$ (kPa) on the pressure drop ΔP (kPa) in a vortex tube for $\mu = 0.63$; 1) $\bar{L} = 4$; 2) 6; 3) 9; 4) 11.

The h.f. flow instability is apparently a result of the formation of large-scale coherent vortex structures of shear nature in the vortex tube exactly as occurs in plane turbulent jet mixing layers with different flow velocities [7]. The process of large-scale vortex structure formation in a helical flow can be represented as follows. The characteristic feature of the flow in a vortex tube is the presence of a discontinuity in the axial velocity component on the interface between the peripheral and stimulated streams. The maximum level of V_z in both streams and therefore the maximum value of the shear stresses also are observed in direct proximity to the nozzle section [1]. The formation of fine-scale shear vortex structures also occurs here, which in the case of complete flow symmetry should be in the shape of a torus encircling the stimulated stream and rotating together with it around the tube axis. In a real turbulent stream, the shear vortex structures rotate in spiral vortex filaments. As they move toward the choke, they merge in pairs to form large-scale vortex structures which are enlarged in turn, etc. The acts of pairwise enlargement (merger) of the vortices will occur until a small number of spiral vortex structures (bunches) remains in the intermediate layer between the peripheral and stimulated streams, whose merger is possible only in the case of disturbance of the circumferential symmetry of their arrangement around the near-axis flow. Such asymmetry can be initiated by a rise in the magnitude of the shear stress because of the increase in ΔP or μ.

Vortex structures regularly perturb the peripheral stream in their translational–rotational motion. The pressure transducer mounted on the tube wall perceives these perturbations as pressure fluctuations of discrete frequency. The amplitude of these pulsations will be determined by the size of the structures, and the frequency by their quantity (n) and angular rotation (ω) around the tube axis ($f_{h.f.} = n\omega/2\pi$). At the instant of merger of pairs of spiral structures, as ΔP increases, say, a smaller quantity of larger-scale vortices is formed, i.e., the pressure transducer should pick up a jump increase in $\Delta P'_{h.f.}$ and a reduction in $f_{h.f.}$ as is indeed obtained in experiment.

Interesting results were obtained in experimental studies devoted to the structure of turbulent rotating streams [2]: the intensity of the radial component of the turbulent fluctuation velocity exceeds the remaining components 5-10 times in a major part of the cross section. On the basis of these data, many researchers assume [2] that turbulence, which is an energy source of elementary cold cycles performed by turbulent elements, underlies the energy exchange mechanism in a vortex tube. The results of our research permit the assertion that large values of the radial velocity fluctuation component are a result of regular radial deformation of the free vortex boundaries by a precessing stimulated vortex and by spiral vortex structures of shear nature, i.e., namely the flow instability observed in the whole range of μ and ΔP values applied in practice generates a directional (radial) periodic pseudoturbulence, and thereby contributes to the radial energy exchange and the realization of the energy separation process as a whole. To comprehend the physical nature of the Ranque effect, a detailed investigation of the dynamic properties of the spiral stream in vortex tubes and a study of their influence on the energy separation process are therefore necessary.

NOTATION

L, d, vortex tube length and diameter; d_d, diaphragm diameter; L_d, distance between the tangential inlet channels and the location of the pressure transducer; F_{in}, area of the tangential inlet channels; ΔP, pressure drop in the vortex tube; $\Delta P'_{h.f.}$, $\Delta P_{l.f.}$, $\Delta P'_{\Sigma}$, amplitudes of the low- and high-frequency pressure fluctuations and the total pressure fluctuation level; $f_{l.f.}$, $f_{h.f.}$, low- and high-frequency pressure fluctuation frequencies; μ, relative mass flow rate of the cooled air; and ΔT_x, ΔT_g, air cooling and heating effects in the vortex tube.

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TURBULENT FLUID FLOW IN A CIRCULAR PIPE WITH UNIFORM BLOWING THROUGH POROUS WALLS

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UDC 532.542

The average and fluctuating incompressible fluid flow characteristics in a circular pipe with blowing are computed on the basis of a three-parameter model of turbulence.

Investigations of flow in channels with permeable walls are of interest for the analysis of heat- and mass-transfer processes in heat pipes when using blowing in the interest of heat shielding and in many other applications. Computations of the turbulent flow in pipes with blowing have been performed in [1, 2] on the basis of mixing-path length models, in [3] for the transition flow mode, and in [4] for the hydrodynamically stabilized stream by using additional equations for the fluctuating motion. Flow development along the pipe length is investigated in this paper for relatively high Reynolds number of the main stream at the input for conditions that are almost realized in experiments [5].

Solutions for the equations of average and fluctuating motion have been obtained in the boundary-layer theory approximation valid for $m \ll 1$. The fluctuating motion is described by a three-parameter model of turbulence, consisting of the equations of fluctuating energy balance, turbulent tangential stresses, and turbulent energy dissipation, described in a form close to the models proposed earlier in [6, 7]. The system of equations used in the computations for axisymmetric stationary incompressible fluid flow in a circular pipe has the form

$$\frac{\partial (ru_x)}{\partial x} + \frac{\partial (ru_r)}{\partial r} = 0, \tag{1}$$

$$u_x \frac{\partial u_x}{\partial x} + u_r \frac{\partial u_x}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\nu \frac{\partial u_x}{\partial r} - \sigma\right)\right], \tag{2}$$

$$u_x \frac{\partial E}{\partial x} + u_r \frac{\partial E}{\partial r} = -\sigma \frac{\partial u_x}{\partial r} - \frac{cE^{3/2}}{L} - \frac{c_{1E}\nu E}{L^2} + \frac{1}{r}\frac{\partial}{\partial r}\left[r(\nu + \alpha_E E^{1/2} L)\frac{\partial E}{\partial r}\right], \tag{3}$$

G. M. Krzhizhanovskii Power Institute, Moscow. Translated from Izhenerno-Fizicheskii Zhurnal, Vol. 41, No. 5, pp. 791-795, November, 1981. Original article submitted September 16, 1980.